Multiplication, Division, and Area
Math in Focus

Unit 2 Curriculum Guide
November 12th- February 1st

ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

Table of Contents

Unit Overview

Unit 2: Chapters 6 – 9 &

Eureka Math Module 4:

Multiplication and Area

In this Unit Students will

- Solve multiplication and division word problems within 100.
- Write an equation to represent a multiplication or division word problem with a symbol for the unknown.
- Draw a visual representation (array, drawing, area model, etc.) for a given multiplication or division word problem.
- Choose the appropriate operation based on context clues in text.
- Fluently multiply and divide within 100 (know from memory all product of two one-digit numbers).
- Describe the relationship between factors and products in terms of multiplication and division.
- Solve the area of a rectangle by tiling and then counting the number of unit squares. Describe the relationship between counting the number of unit squares and multiplying the side lengths in finding the area of a rectangle.
- Solve the area of a rectangle by multiplying its side lengths.
- Solve real-world area problems by either tiling or multiplying the side lengths.
- Solve for the area of a rectangle by breaking one side into a sum (example if the length is 5 break it apart as 2 + 3) then multiplying each part/addend by the other side.
- Explain why the two strategies above produce the same area (proving distributive property).
- Add square units to find the area of a given shape by counting the square of the visual.
- Multiply length times (x) width to find the area of a given shape.
- Find the area of a rectilinear figure and add the non-overlapping parts/units.

Essential Questions (Bold Writing= Largely Suggested)

- What stories or situations can be expressed as 5x 7?
- In what context can a number of groups or a number of shares can be expressed as 56÷8?
- What kinds of problems in your word might be modeled and solved with multiplication/division?
- How might multiplication help you solve a division problem?
- How can estimation be useful when solving multiplication and division problems?

- What strategies can you utilized to solve an unknown fact?
- How can you use know facts to help you find unknown facts? If you don't know 6x9, how can you use 6x10 to help?
- What properties help you solve an unknown fact or unknown products?
- How can you explain the patterns observed in multiplication and division combinations/facts?
- Why does "what" we measure influence "how" we measure?
- What units and tools are used to measure?
- How are multiplication and addition alike or related?
- How are multiplication and addition different?
- What are strategies for learning multiplication facts?
- How can we practice multiplication facts in a meaningful way that will help us remember them?
- How can we connect multiplication facts with their array models?
- How is the commutative property of multiplication evident in an array model?
- What patterns of multiplication can we discover by studying a times table chart?
- How can we determine numbers that are missing on a times table chart by knowing multiplication patterns?
- What role can arithmetic properties play in helping us understand number patterns?
- How can we model multiplication?
- How can we write a mathematical sentence to represent a multiplication model we have made?
- Is there more than one way of multiplying to get the same product?
- What patterns can be found when multiplying numbers?
- What pattern is there when we multiply by ten or a multiple of ten? By one? By zero?
- How can multiplication help us repeatedly add larger numbers?
- How does the order of the digits in a multiplication problem affect the product? How is area related to multiplication?
- What is the area of this 4 by 6-inch figure? Prove your answer by using both addition and multiplication.
- Why does multiplying side length determine the area of a rectangle? Will it always work?
- How can you decompose this figure to identify its area?
- How can you use the side lengths of this figure that are given to determine the side lengths that are not given?

Enduring Understandings

- Multiplication can be thought of as repeated addition.
- Multiplication facts can be deduced from patterns.
- The **associative property of multiplication** can be used to simplify computation. The **associative property of multiplication** is – when I multiply 3 numbers, the way the numbers are grouped does not change the product.
- The **distributive property of multiplication** allows us to find partial products and then find their sum. The **distributive property** is – when I multiply the sum of 2 numbers by a 3rd number, it is the same as multiplying each addend by the 3rd number and adding the product.
- Patterns are evident when multiplying a number by ten or a multiple of ten.
- Multiplication and division are inverses; they undo each other.
- Multiplication and division can be modeled with arrays.
- Multiplication is commutative, but division is not. The **commutative property of multiplication** is - the order of the factors does not change the product.
- There are **two common situations where division** may be used.
    - **Partition** (or fair-sharing) - given the total amount and the number of equal groups, determine how many/much in each group **PARTITVE**
    - **Measurement** (or repeated subtraction) - given the total amount and the amount in a group, determine how many groups of the same size can be created. **QUOTATIVE**

- There is a relationship between the divisor, the dividend, the quotient, and any remainder. Students recognize area as an attribute of two-dimensional shape.
- As the divisor increases, the quotient decreases; as the divisor decreases, the quotient increases.
- Students measure the area of a shape by finding the total number of same size units of area required to cover the shape without gaps or overlaps.
- Students understand that rectangular arrays can be decomposed into identical rows or into identical columns.
- By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.
- Students find area in square units such as square inches, square meters, square centimeters, and square feet
- Students analyze the relationship between the units used to find area and perimeter
- Students apply what they know about area and perimeter to solve real-world problems
- Use the formula correctly to find the area and perimeter of rectangle and square
    - Solve word problems involving area and perimeter of rectangles and squares

MIF Pacing Guide

Activity	Common Core Standards	Time
6.1 Multiplication Properties with Problem Solving	3.OA 4 -7	2 blocks
6.2 Multiply by 6 with Problem Solving	3.OA 4 -7, 3.OA.9	1 block
6.3 Multiply by 7 with Problem Solving	3.MD. 6 and 3.OA.9	1 block
6.4 Multiply by 8 with Problem Solving	3.MD. 6 and 3.OA.9	1 block
6.5 Multiply by 9 with Problem Solving	3.MD. 6 and 3.OA.9	1 block
6.6 Division: Finding the Number of Items in Each Group with Problem Solving	3.OA,2-3 and 3.OA.6	1 block
6. 7 Division: Making Equal Groups with Problem Solving	3.OA,2 and 3.OA.6-7	1 block
Chapter 6 Test/Performance Task	3.OA.4-7, 3.OA.9, 3.MD.6	1 block
7.1 Mental Multiplication with Problem Solving	3.NBT.3, 3.OA3-5, 3.OA.7 3.OA.9	1 block
7.2 Multiplying without Regrouping	3.OA.4, 3.OA.5, 3.OA.7	1 block
7.3 Multiplying Ones, Tens, and Hundreds with Regrouping	3.NBT.3, 3.OA.4, 3.OA.5, 3.OA.7	2 blocks
Chapter 7 Test/Performance Task	3.NBT.3, 3.OA3-5, 3.OA.7 3.OA.9	1 block
8.1 Mental Division with Problem Solving	3.OA4-7	1 block
8.2 Quotient and Remainder	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7, 3.OA.9	1 block
8.3 Odd and Even Numbers	3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7, 3.OA.9	1 block
8.4 Division without Remainder and Regrouping	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7	1 block
8.5 Division with Regrouping in Tens and Ones	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.6, 3.OA.7	1 block
Chapter 8 Test/Performance Task	3.OA4-7, 3.OA.9	1 block
9.1 Real-World Problems: Multiplication	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7	1 block
9.2 Real-World Problems: Two-Step Problems with Multiplication	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.8	2 blocks
9.3 Real-World Problems: Division	3.OA.3, 3.OA.6	1 block
9.4 Real-World Problems: Two-Step Problems with Division	3.OA.3, 3.OA.6, 3.OA.8	3 blocks
Chapter 9 Test/Performance Task	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.8	1 block

Eureka Math Module 4: Multiplication and Area		
Topic	**Lesson**	**Lesson Objective/ Supportive Videos**
	Lesson	Understand area as an attribute of plane figures.

Topic A: Foundations for Understanding Area	1	https://www.youtube.com/watch?v
	Lesson 2/3	Decompose and recompose shapes to compare areas. Model tiling with centimeter and inch unit squares as a strategy to measure area. https://www.youtube.com/watch?v
	Lesson 4	Relate side lengths with the number of tiles on a side. https://www.youtube.com/watch?v
Topic B: Concepts of Area Measurement	Lesson 5	Form rectangles by tiling with unit squares to make arrays. https://www.youtube.com/watch?v
	Lesson 6	Draw rows and columns to determine the area of a rectangle, given an incomplete array. https://www.youtube.com/watch?v
	Lesson 7	Interpret area models to form rectangular arrays. https://www.youtube.com/watch?v
	Lesson 8	Find the area of a rectangle through multiplication of the side lengths. https://www.youtube.com/watch?v
Topic C: Arithmetic Properties Using Area Models	Lesson 10	Apply the distributive property as a strategy to find the total area of a large rectangle by adding two products. https://www.youtube.com/watch?v
	Lesson 11	Demonstrate possible whole number side lengths of rectangles with areas of 24, 36, 48, or 72 square units using the associative property. https://www.youtube.com/watch?v
Topic D: Applications of Area Using Side Lengths of Figures	Lesson 12	Solve word problems involving area. Solve word problems involving area. https://www.youtube.com/watch?v
	Lesson 13	Find areas by decomposing into rectangles or completing composite figures to form rectangles. https://www.youtube.com/watch?v
	Lesson 14	Find areas by decomposing into rectangles or completing composite figures to form rectangles. https://www.youtube.com/watch?v

Common Core State Standards	
3.OA.1	Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*

Students recognize multiplication as a means to determine the total number of objects when there are a specific number of groups with the same number of objects in each group. Multiplication requires students to think in terms of groups of things rather than individual things. Students learn that the multiplication symbol 'x' means "groups of" and problems such as 5 x 7 refer to 5 groups of 7.

To further develop this understanding, students interpret a problem situation requiring multiplication using pictures, objects, words, numbers, and equations. Then, given a multiplication expression (e.g., 5 x 6) students interpret the expression using a multiplication context. They should begin to use the terms, *factor* and *product,* as they describe multiplication.

For example: Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase? 5 groups of 3, 5 x 3 = 15. Describe another situation where there would be 5 groups of 3 or 5 x 3.

Sonya earns $7 a week pulling weeds. After 5 weeks of work, how much has Sonya worked? Write an equation and find the answer. Describe another situation that would match 7 x 5.

3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*

Students recognize the operation of division in two different types of situations. One situation requires determining how many groups and the other situation requires sharing (determining how many in each group). Students should be exposed to appropriate terminology (quotient, dividend, divisor, and factor).

To develop this understanding, students interpret a problem situation requiring division using pictures, objects, words, numbers, and equations. Given a division expression (e.g., 24 ÷ 6) students interpret the expression in contexts that require both interpretations of division.

For example: Partition models provide students with a total number and the number of groups. These models focus on the question, "How many objects are in each group so that the groups are equal?" A context for partition models would be: There are 12 cookies on the counter. If you are sharing the cookies equally among three bags, how many cookies will go in each bag?

Measurement (repeated subtraction) models provide students with a total number and the number of objects in each group. These models focus on the question, "How many equal groups can you make?' A context for measurement models would be: There are 12 cookies on the counter. If you put 3 cookies in each bag, how many bags will you fill?

3.OA.3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.

This standard references various problem solving context and strategies that students are expected to use while solving word problems involving multiplication and division. Students should use variety of representations for creating and solving one step word problems, such as: If you divide 4 packs of 9 brownies among 6 people, how many brownies does each person receive? (4 x 9 = 36, 36 ÷ 6 = 9).

It is important for students to have many opportunities to use concrete materials to model the situations and identify the number of groups and the number of items in a group.

3.OA.4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = □÷ 3, 6 × 6 = ?.*

This standard is strongly connected to 3.OA.3 where students solve problems and determine unknowns in equations. Students should also experience creating story problems for given equations. When crafting story problems, they should carefully consider the question(s) to be asked and answered to write an appropriate equation. Students may approach the same story problem differently and write either a multiplication equation or division equation.

Students apply their understanding of the meaning of the equal sign as “the same as” to interpret an equation with an unknown. When given 4 x ? = 40, they might think:

- 4 groups of some number is the same as 40
- 4 times some number is the same as 40
- I know that 4 groups of 10 is 40 so the unknown number is 10
- The missing factor is 10 because 4 times 10 equals 40.

Equations in the form of a x b = c and c = a x b should be used interchangeably, with the unknown in different positions.

Examples:

- Solve the equations below:
  24 = ? x 6
  72 ÷ Δ = 9
- Rachel has 3 bags. There are 4 marbles in each bag. How many marbles does Rachel have altogether? 3 x 4 = *m*

Students may use interactive whiteboards to create digital models to explain and justify their thinking.

3.OA.5	Apply properties of operations as strategies to multiply and divide. Examples: If 6 x 4 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 x 10 = 30 (Associative property of multiplication) Knowing that 8 x 5 = 40 and 8 x 2 = 16, one can find 8 x 7 as 8 x (5 + 2) = (8 x 5) + (8 x 2) = 40 + 16 = 56 (Distributive property)

As third graders explore and develop conceptual understanding of multiplication and division, they recognize the structure of multiplication by noticing patterns and making generalizations about multiplication and division applying a variety of properties. These properties are not taught in isolation, but rather should be developed and discussed as a part of the carefully related student experiences. Incorporate opportunities for students to use the properties to develop strategies and patterns.

Providing students with multiple experiences to multiply with a factor of 1 will lead to the discussion of 1 as the identity element for multiplication.

The zero property of multiplication states that if one of the factors is zero the product is zero. Although the product for 6 x 3 and 3 x 6 is the same, the actual multiplication situations are not the same. One represents 6 groups of 3 and the other represents 3 groups of 6. The associative property of multiplication shows that when multiplying 3 or more numbers, the product is always the same regardless of the grouping. The distributive property should be explored in the context of composing and decomposing factors.	
3.OA.6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.
Thinking about division in terms of multiplication will help students to use the multiplication facts they know to become fluent with division facts.	
3.OA.7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
By studying patterns and relationships in multiplication facts and relating multiplication and division, students build a foundation for fluency with multiplication and division facts. Students demonstrate fluency with multiplication facts through 10 and the related division facts. Multiplying and dividing fluently refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently. Strategies students may use to attain fluency include: • Multiplication by zeros and ones • Doubles (2s facts), Doubling twice (4s), Doubling three times (8s) • Tens facts (relating to place value, 5 x 10 is 5 tens or 50) • Five facts (half of tens) • Skip counting (counting groups of __ and knowing how many groups have been counted) • Square numbers (ex: 3 x 3) • Nines (10 groups less one group, e.g., 9 x 3 is 10 groups of 3 minus one group of 3) • Decomposing into known facts (6 x 7 is 6 x 6 plus one more group of 6) • Turn-around facts (Commutative Property) • Fact families (Ex: 6 x 4 = 24; 24 ÷ 6 = 4; 24 ÷ 4 = 6; 4 x 6 = 24) • Missing factors General Note: Students should have exposure to multiplication and division problems presented in both vertical and horizontal forms.	
3.OA.8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
Students should be exposed to multiple problem-solving strategies (using any combination of words, numbers, diagrams, physical objects or symbols) and be able to choose which ones to use. When students solve word problems, they use various estimation skills which include identifying when estimation is appropriate, determining the level of accuracy needed, selecting the appropriate method of estimation, and verifying solutions or determining the reasonableness of solutions.	

Student 1	Student 2	Student 3
I first thought about 267 and 34. I noticed that their sum is about 300. Then I knew that 194 is close to 200. When I put 300 and 200 together, I get 500.	I first thought about 194. It is really close to 200. I also have 2 hundreds in 267. That gives me a total of 4 hundreds. Then I have 67 in 267 and the 34. When I put 67 and 34 together that is really close to 100. When I add that hundred to the 4 hundred that I already had, I end up with 500.	I rounded 267 to 300. I rounded 194 to 200. I rounded 34 to 30. When I added 300, 200, and 30. I know my answer be about 500.

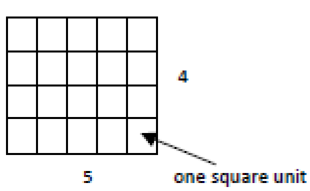
3.OA.9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.

Identifying and explaining patterns leads students to develop the ability to make generalizations, which is the foundation of algebraic reasoning and more formal mathematical thinking.

Students should have opportunities to explore, recognize, and talk about patterns in addition, subtraction, multiplication, and division. Students should be expected to justify their thinking.

3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

These standards call for students to explore the concept of covering a region with "unit squares," which could include square tiles or shading on grid or graph paper. Based on students' development, they should have ample experiences filling a region with square tiles before transitioning to pictorial representations on graph paper.



3.MD.6	Measure areas by counting unit squares (squares cm, square m, square in, square ft, and improvised units).

Students should be counting the square units to find the area could be done in metric, customary, or non-standard square units. Using different sized graph paper, students can explore the areas measured in square centimeters and square inches. The task shown above would provide great experiences for students to tile a region and count the number of square units.	
3.MD.7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real-world and mathematical problems, and represent whole-number products as rectangular area in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side length *a* and *b + c* is the sum of *a x b* and *a x c*. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-over-lapping parts, applying this technique to solve real word problems.
Students can learn how to multiply length measurements to find the area of a rectangular region. But, in order that they make sense of these quantities, they must first learn to interpret measurement of rectangular regions as a multiplicative relationship of the number of square units in a row and the number of rows. This relies on the development of spatial structuring. To build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows, students might draw rectangular arrays of squares and learn to determine the number of squares in each row with increasingly sophisticated strategies, such as skip-counting the number in each row and eventually multiplying the number in each row by the number of rows. They learn to partition a rectangle into identical squares by anticipating the final structure and forming the array by drawing line segments to form rows and columns. They use skip counting and multiplication to determine the number of squares in the array. Many activities that involve seeing and making arrays of squares to form a rectangle might be needed to build robust conceptions of a rectangular area structured into squares. Students should understand and explain why multiplying the side lengths of a rectangle yields the same measurement of area as counting the number of tiles (with the same unit length) that fill the rectangle's interior. For example, students might explain that one length tells how many unit squares in a row and the other length tells how many rows there are. Students should tile rectangle then multiply the side lengths to show it is the same.	

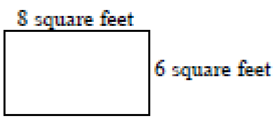
To find the area one could count the squares or multiply 3 x 4 = 12.

1	2	3	4
5	6	7	8
9	10	11	12

Students should solve real world and mathematical problems.

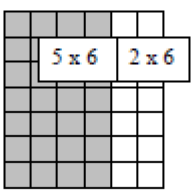
Example:

Drew wants to tile the bathroom floor using 1 foot tiles. How many square foot tiles will he need?



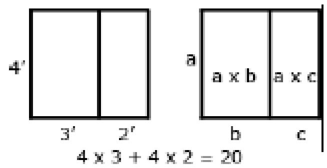
Students might solve problems such as finding all the rectangular regions with whole-number side lengths that have an area of 12 area units, doing this for larger rectangles (e.g. enclosing 24, 48, 72 area-units), making sketches rather than drawing each square. Students learn to justify their belief they have found all possible solutions.

This standard extends students' work with distributive property. For example, in the picture below the area of a 7 x 6 figure can be determined by finding the area of a 5 x 6 and 2 x 6 and adding the two sums.



Using concrete objects or drawings students build competence with composition and decomposition of shapes, spatial structuring, and addition of area measurements, students learn to investigate arithmetic properties using area models. For example, they learn to rotate rectangular arrays physically and mentally, understanding that their area are preserved under rotation, and thus for example, 4 x 7 = 7 x 4, illustrating the commutative property of multiplication. Students also learn to understand and explain that the area of a rectangular region of, for example, 12 length-units by 5 length-units can be found either by multiplying 12 x 5, or by adding two products, e.g. 10 x 5 and 2 x 5, illustrating distributive property.

Example:

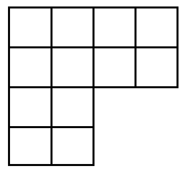


4 x 3 + 4 x 2 = 20

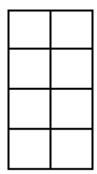
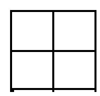
4 (3 + 2) = 20

4 x 5 = 20

This standard uses the word rectilinear. A rectilinear figure is a polygon that has all right angles.



How could this figure be decomposed to help find the area?

 This portion of the decomposed figure is a 4 x 2.	 This portion of the decomposed figure is 2 x 2.

4 x 2 = 8 and 2 x 2 = 4

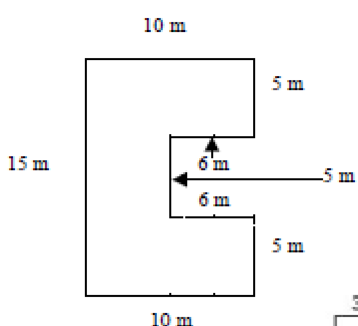
So 8 + 4 = 12

Therefore the total area of this figure is 12 square units

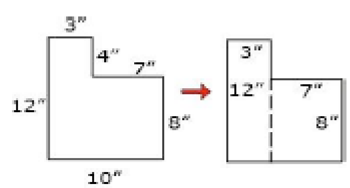
Example:

A storage shed is pictured below. What is the total area?

How could the figure be decomposed to help find the area?



Example:
Students can decompose a rectilinear figure into different rectangles. They find the area of the figure by adding the areas of each of the rectangles together.



area is 12 x 3 + 8 x 7 = 92 sq inches

3.NBT.3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.
Students begin using concrete models and groups of 10. Number lines and skip counting support student thinking. It is important that students have many experiences to model, explain, and generalize rather than be shown tricks about adding zeros. Such tricks and short cuts hinder understanding of place value's role in multiplication.	
M : Major Content S: Supporting Content A : Additional Content	

MIF Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	**Chapter Opener** Assessing Prior Knowledge *The Pre Test serves as a diagnostic test of readiness of the upcoming chapter*	**Teacher Materials** Quick Check Pretest (Assessm't Bk) Recall Prior Knowledge **Student Materials** Student Book (Quick Check); Copy of the Pre Test; Recall prior Knowledge	Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test ; Students write Quick Check answers on a separate sheet of paper Quick Check and the Pre Test can be done in the same block *(See Anecdotal Checklist; Transition Guide)* Recall Prior Knowledge – Quick Check – Pre Test
DIRECT ENGAGEMENT	**Direct Involvement/Engagement** Teach/Learn *Students are directly involved in making sense, themselves, of the concepts – by interacting the tools, manipulatives, each other, and the questions*	**Teacher Edition** 5-minute warm up Teach; Anchor Task **Technology** Digi **Other** Fluency Practice	• The Warm Up activates prior knowledge for each new lesson • Student Books are CLOSED; Big Book is used in Gr. K • Teacher led; Whole group • Students use concrete manipulatives to explore concepts • A few select parts of the task are explicitly shown, but the majority is addressed through the hands-on, constructivist approach and questioning • Teacher facilitates; Students find the solution
GUIDED LEARNING	**Guided Learning and Practice** Guided Learning	**Teacher Edition** Learn **Technology** Digi **Student Book** Guided Learning Pages Hands-on Activity	Students-already in pairs /small, homogenous ability groups; Teacher circulates between groups; Teacher, anecdotally, captures student thinking **Small Group w/Teacher circulating among groups** Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	**Independent Practice** *A formal formative assessment*	**Teacher Edition** Let's Practice **Student Book** Let's Practice **Differentiation Options** All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	**Let's Practice** determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice. Manipulatives **CAN** be used as a communications tool as needed. Completely Independent On level/advance learners should finish all workbook pages.
ADDITIONAL PRACTICE	**Extending the Lesson**	Math Journal Problem of the Lesson Interactivities Games	
	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	**End of Chapter Wrap Up and Post Test**	**Teacher Edition** Chapter Review/Test Put on Your Thinking Cap **Student Workbook** Put on Your Thinking Cap **Assessment Book** Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed • Individually (e.g. for homework) then reviewed in class • As a 'mock test' done in class and doesn't count • As a formal, in class review where teacher walks students through the questions Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February, students should be doing the Put on Your Thinking Cap problems on their own.

Math Background

During their elementary mathematics education, students learned basic meanings of equal groups for multiplication and division. They were taught that multiplication is the addition of equal groups, division as sharing equally. They were also taught to group items equally, as well as how to relate and apply the concepts to word problems.

Students learned multiplication as repeated addition, and division as sharing or grouping. Also, they learned the multiplication facts for 2, 3, 4, 5, and 10. They also learned that multiplication and division are related as inverse operations. So, division can be used to find a missing factor. The missing number in the number sentence
2 x __ = 8 is the answer to 8 divided by 2.

Students learned to use bar models to solve two-step problems involving addition and subtraction. This is extended in this unit to include multiplication and division. During their elementary mathematics education, students learned to multiply using an area model in Unit 2. This crucial skill enables students to multiply efficiently in order to find area of figures. Students learned about the basic shapes such as circles, triangles, and squares, to name a few. Students learned that combining or decomposing plane shapes produced other shapes. Students were taught to identify plane shapes and this enables them to subsequently find the area and perimeter of figures.

Misconceptions

- Students may not understand story problems. Maintain student focus on the meaning of the actions and number relationships, and encourage them to model the problem or draw as needed. Students often depend on key words, strategy that often is not effective. For example, they might assume that the word left always means that subtraction must be used. Providing problems in which key words are used to represent different operations is essential. For example, the use of the word left in this problem does not indicate subtraction. Suzy took 28 stickers she no longer wanted and gave them to Anna. Now Suzy has 50 stickers left. How many stickers did Suzy have to begin with? Students need to analyze word problems and avoid using key words to solve them.

- Students may not interpret multiplication by considering one factor as the number of groups and the other factor as the number in each group. Have student model multiplication situations with manipulatives or pictorially. Have students write multiplication and division word problems.

- Students solve multiplication word problems by adding or dividing problems by subtracting. Students need to consider whether a word problem involves taking apart or putting together equal groups. Have students model word problems and focus on the equal groups that they see.

- Students believe that you can use the commutative property for division. For example, students think that 3 ÷ 15 =15 is the same as 15 ÷ 3 =5. Have students represent the problem using models to see the difference between these two equations. Have them investigate division word problems and understand that division problems give the whole and an unknown, either the number of groups or the number in each group.
- Students may not understand the relationship between addition/multiplication and subtraction/division. Multiplication can be understood as repeated addition of equal groups; division is repeated subtraction of equal groups. Provide students with word problems and invite students to solve them. When students solve multiplication problems with addition, note the relationship between the operations of addition and multiplication and the efficiency that multiplication offers. Do the same with division problems and subtraction.

- Students may not understand the two types of division problems**. Division problems are of two different types--finding the number of groups** ("**quotative**" or "**measurement"**) and finding the number in each group ("**partitive/equally sharing/fairly dealing** " or "/**quotative/equally grouping/repeatedly subtracting**"). Make sure that students solve word problems of these two different types. Have them create illustrations or diagrams of each type and discuss how they are the same and different. Connect the diagrams to the equations.

- Students use the addition, subtraction, multiplication, or division algorithms incorrectly. Remember that the traditional algorithms are only one strategy. Partial sums, partial products, and partial quotients are examples of alternative strategies that highlight place value and properties of operations, have students solve problems using multiple models, including numbers, pictures, and words.

- Students think a symbol (? or []) is always the place for the answer. This is especially true when the problem is written as 15 ÷ 3 =? or 15 = o x 3. Students also think that 3 ÷ 15 = 5 and 15 ÷ 3 = 5 are the same equations. The use of models is essential in helping students eliminate this understanding. The use of a symbol to represent a number once cannot be used to represent another number in a different problem/situation. Presenting students with multiple situations in which they select the symbol and explain what it represents will counter this misconception.

- Students confuse area and perimeter. Introduce the ideas separately. Create real word connections for these ideas. For example, the perimeter of a white board is illustrated by the metal frame. The area of the floor is illustrated by the floor tiles. Use the vocabulary of area and perimeter in the context of the school day. For example, have students sit on the "perimeter" of the rug.

- Students may have difficulty using know side lengths to determine unknown side lengths. Offer these students identical problems on grip paper and without the gridlines. Have them compare the listed length to the gridlines that the lines represent. Transition students to problems without gridlines, but have grid paper available for students to use to confirm their answers.

- Students may use formulas to find area and perimeter but may not understand the connection.

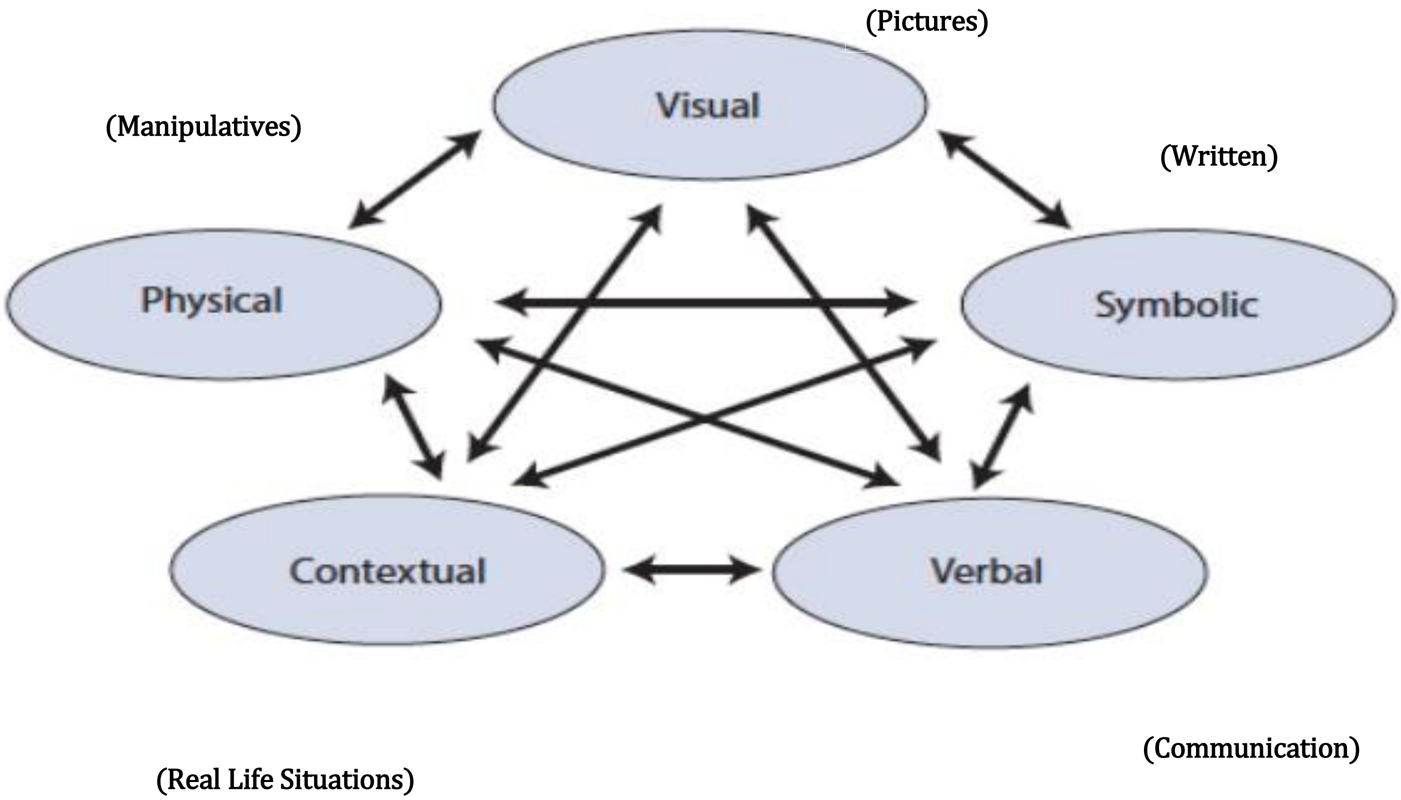
PARCC Assessment Evidence/Clarification Statements

NJSLS	Evidence Statement	Clarification	Math Practices
3.OA.1	Interpret products of whole numbers, e.g. interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 x 7.*	i) Task involve interpreting products in terms of equal groups, arrays, area, and/or measurement quantities. ii) Tasks do not require students to interpret products in terms of repeated addition, skip-counting, or jumps on the number line. iii) The italicized example refers to describing a context. But describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a total can be expressed as a specified product.	4,2
3.OA.2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects. *For example, describe a context in which a number shares or a number of groups can be expressed as 56÷8.*	i) Tasks involve interpreting quotients in terms of equal groups, arrays, area, and/or measurement quantities. ii) Tasks do not require students to interpret quotients in terms of repeated addition, skip-counting, or jumps on the number line. iii) The italicized example refers to describing a context. But describing a context is not the only way to meet the standard. For example, another way to meet the standard would be to identify contexts in which a total can be expressed as a specified product. iv) 50% of tasks require interpreting quotients as a number of objects in each share. 50% of tasks require interpreting quotients as a number of equal shares.	4,2
3.OA.3-1	Use multiplication within 100 to solve word problems in situations involving equal groups, arrays, or area, e.g by using drawings and equations with a symbol for the unknown number to represent the problems.	i) All problems come from the harder three quadrants of the times table (ax b, where a > 5 and/or b >5). ii) 50% of task involve multiplying to find the total number (equal groups, arrays); 50% involve multiplying to find the area. iii) For more information see CCSS Table 2 page 89 and the Progression document for Operations and Algebraic Thinking.	1,4
3.OA.3-2	Use multiplication within 100 to solve word problems in situations involving equal groups, arrays, or area, e.g by using drawings and equations with a symbol for the	i) All problems come from the harder three quadrants of the times table (a x b, where a > 5 and/or b > 5). ii). Tasks involve multiplying to find a total measure (other than area).	1,4

	unknown number to represent the problems.	iii). For more information see CCSS Table 2 page 89 and the Progression document for Operations and Algebraic Thinking.	
3.OA.3-3	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve word problems in situations involving equal groups, arrays, or area, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.	i). All quotients are related to products from the harder three quadrants of the times table (a x b where a > 5 and/or b > 5). ii). A third of tasks involve dividing to find the number in each equal group or in each equal row/column of an array; a third of tasks involve dividing to find the number of equal groups or the number of equal rows/columns of an array; a third of task involve dividing an area by a side length to find an unknown side length. iii). For more information see CCSS Table 2 page 89 and the Progression document for Operations and Algebraic Thinking.	1,4
3.OA.3-4	Use division within 100 (quotients related to products having both factors less than or equal to 10) to solve problems in situations involving measurement quantities other than area, e.g. by using drawings and equations with a symbol for the unknown number to represent the problem.	i). All quotients are related to products from the harder three quadrants of the times table (a x b where a > 5 and/or b > 5). ii). 50% of tasks involve finding the number of equal pieces; 50% involve finding the measure of each piece. iii).For more information see CCSS Table 2 page 89 and the Progression document for Operations and Algebraic Thinking.	1, 4
3.OA.4	Determine the unknown whole number in a multiplication or division equation relating the whole numbers. For example, determine the unknown number that makes the equation true in each equation. 8 x ? = 48, 5 = ? + 3, 6 x 6 =?	i) Tasks do not have a context. ii) Only the answer is required (methods, representations, etc...are not assessed here). iii) All products and related quotients are from the harder three quadrants of the times table (a x b where a > 5 and/or b >5).	-
3.OA.6	Understand division as an unknown factor problem. For example find 32÷8 by finding the number that makes 32 when multiplied by 8.	i) All products and related quotients are related to products from the harder three quadrants of the times table (a x b where a > 5 and/or b > 5).	
3.OA. 7	Fluently multiply and divide within 25, using strategies such as the relationship between multiplication and division (e.g. knowing that 4x4 = 16, one knows that 16÷4=4) or properties of operations. By the end of grade 3, know from memory all	i). Tasks do not have a context. ii). Only the answer is required (strategies, representations, etc. are not assessed here). iii). Tasks require fluent (fast and accurate) finding of products and related quotients. For example, each	

	products of two one digit numbers.	one point task might require four or more computations, two or more multiplication, and two or more division. However, tasks are not explicitly timed.	
3.OA.8-1	Solve two-step word problems using the four operations (for Unit 1 just two addition and subtraction) Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	i) Only the answer is required (methods, representations, etc. are not assessed here). iii) Addition, subtraction, multiplication, and division situations in these problems man involve any of the basic situations types with unknowns in various positions. iii) If scaffolded, one of the 2 parts must require 2-steps. The other part may consist of 1-step. iv) Conversions should be part of the 2-steps and should not be a step on its own. v) If the item is 2 points, the item should be a 2 point, unscaffolded item but the rubric should allow for 2-1-0 points.	1, 4
3.MD.5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	None	7
3.MD.6	Measure areas by counting unit squares (squares cm, square m, square in, square ft, and improvised units).	None	7

Use and Connection of Mathematical Representations



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

Visual: When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

Physical: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

Verbal: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

Symbolic: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

Contextual: A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

Concrete: “Doing Stage”: Physical manipulation of objects to solve math problems.

Pictorial: “Seeing Stage”: Use of imaged to represent objects when solving math problems.

Abstract: “Symbolic Stage”: Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

Read, Draw, Write Process

READ the problem. Read it over and over…. And then read it again.

DRAW a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

WRITE your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students their understanding of the context and actions taking place in a problem, how a problem is solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

<u>Teacher Questioning:</u>

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple “yes” or “no,” or do they invite students to deepen their understanding?

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.

100 questions that promote Mathematical Discourse

Help students **work together** to make sense of mathematics

1. What **strategy** did you use?
2. Do you **agree**?
3. Do you **disagree**?
4. Would you **ask the rest of the class** that question?
5. Could you **share your method** with the class?
6. What part of what he said **do you understand**?
7. Would someone like to **share** ___?
8. Can you **convince the rest of us** that your answer makes sense?
9. **What do others think** about what [student] said?
10. Can someone **retell or restate** [student]'s explanation?
11. Did you **work together**? In what way?
12. Would anyone like to **add to what was said**?
13. Have you **discussed** this with your group? With others?
14. Did anyone get a **different answer**?
15. **Where** would you go for **help**?
16. **Did everybody get a fair chance** to talk, use the manipulatives, or be the recorder?
17. How could you help another student **without telling them the answer**?
18. **How would you explain** ___ to someone who missed class today?

Help students **rely more on themselves** to determine whether something is mathematically correct

19. Is this a **reasonable answer**?
20. Does that make **sense**?
21. **Why** do you think that? Why is that true?
22. Can you **draw a picture or make a model** to show that?
23. **How** did you reach that conclusion?
24. Does anyone want to **revise** his or her answer?
25. **How were you sure** your answer was right?

Ready

Help students learn to **reason mathematically**

26 **How did you begin** to think about this problem?

27 What is **another way** you could solve this problem?

28 How could you **prove** ______?

29 Can you **explain how your answer is different from or the same as** [student]'s answer?

30 Let's **break the problem into parts**. What would the parts be?

31 Can you **explain this part more specifically**?

32 Does that **always work**?

33 Can you think of a case where that **wouldn't work**?

34 How did you **organize** your information? Your thinking?

Help students **evaluate their own processes** and engage in productive peer interaction

35 What do you need to do **next**?

36 What have you **accomplished**?

37 What are your **strengths and weaknesses**?

38 Was your **group participation appropriate and helpful**?

Help students with **problem comprehension**

39 What is this problem about? What can you **tell me about it**?

40 Do you need to **define or set limits** for the problem?

41 How would you **interpret** that?

42 Could you **reword that in simpler terms**?

43 Is there something that can be **eliminated** or that is **missing**?

44 Could you **explain** what the problem is asking?

45 What **assumptions** do you have to make?

46 What do you **know** about this part?

47 Which words were **most important**? Why?

Help students learn to **conjecture, invent, and solve** problems

48 **What would happen if ___?**

49 Do you see a **pattern**?

50 What are some **possibilities** here?

51 Where could you find the **information** you need?

52 How would you **check your steps** or your answer?

53 What **did not work**?

54 How is your solution method the **same as or different from** [student]'s method?

55 Other than retracing your steps, **how can you determine** if your answers are appropriate?

56 How did you **organize** the information? Do you have a **record**?

57 How could you solve this using **tables, lists, pictures, diagrams**, etc.?

58 What have you tried? What **steps** did you take?

59 How would it look if you used this **model** or these **materials**?

60 How would you draw a **diagram or make a sketch** to solve the problem?

61 Is there **another possible answer**? If so, explain.

62 Is there **another way to solve** the problem?

63 Is there **another model** you could use to solve the problem?

64 Is there anything you've **overlooked**?

65 **How did you think** about the problem?

66 What was your **estimate or prediction**?

67 How **confident** are you in your answer?

68 **What else** would you like to know?

69 What do you think comes **next**?

70 Is the solution **reasonable**, considering the context?

71 Did you have a **system**? Explain it.

72 Did you have a **strategy**? Explain it.

73 Did you have a **design**? Explain it.

Help students learn to **connect mathematics, its ideas, and its application**

74 What is the **relationship** between ___ and ___?

75 Have we ever solved a problem **like this before**?

76 What uses of mathematics did you find in the **newspaper** last night?

77 What is the **same**?

78 What is **different**?

79 Did you use skills or build on concepts that were **not necessarily mathematical**?

80 Which **skills or concepts** did you use?

81 What **ideas** have we explored before that were useful in solving this problem?

82 Is there a **pattern**?

83 **Where else** would this strategy be useful?

84 How does this **relate** to ___?

85 Is there a **general rule**?

86 Is there a **real-life situation** where this could be used?

87 How would your method work with **other problems**?

88 What other problem does this seem to **lead to**?

Help students **persevere**

89 Have you tried making a **guess**?

90 **What else** have you tried?

91 Would **another method** work as well or better?

92 Is there **another way** to draw, explain, or say that?

93 Give me another **related problem**. Is there an easier problem?

94 How would you **explain** what you know right now?

Help students focus on the mathematics from **activities**

95 What was **one thing you learned** (or two, or more)?

96 Did you **notice any patterns**? If so, describe them.

97 What **mathematics topics** were used in this investigation?

98 What were the **mathematical ideas** in this problem?

99 What is mathematically **different about these two situations**?

100 What are the **variables** in this problem? What stays **constant**?

Conceptual Understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

Procedural Fluency

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students’ conceptual understanding of procedures should precede and coincide with instruction on procedures.

Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

3-5 Math Fact Fluency Expectation

3.OA.C.7: Single-digit products and quotients (Products from memory by end of Grade 3)

3.NBT.A.2: Add/subtract within 1000

4.NBT.B.4: Add/subtract within 1,000,000/ Use of Standard Algorithm

5.NBT.B.5: Multi-digit multiplication/ Use of Standard Algorithm

Evidence of Student Thinking

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

Mathematical Proficiency

To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- Strategic competence: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.

(Wiliam 2007, pp. 1054; 1091)

Connections to the Mathematical Practices

Student Friendly Connections to the Mathematical Practices

1. I can solve problems without giving up.
2. I can think about numbers in many ways.
3. I can explain my thinking and try to understand others.
4. I can show my work in many ways.
5. I can use math tools and tell why I choose them.
6. I can work carefully and check my work.
7. I can use what I know to solve new problems.
8. I can discover and use short cuts.

Connections to the Mathematical Practices

1	**Make sense of problems and persevere in solving them**
	In **third** grade, students know that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Third graders may use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?" They listen to the strategies of others and will try approaches. They often will use another method to check their answers.
2	**Reason abstractly and quantitatively**
	In **third** grade, students should recognize that number represents a specific quantity. They connect quantity to written symbols and create logical representation of the problem at hand, considering both the appropriate units involved and the meaning of quantities
3	**Construct viable arguments and critique the reasoning of others**
	In **third** grade, mathematically proficient students may construct viable arguments using concrete referents, such as objects, pictures, and drawings. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like, "How did you get that?" and "Why is it true?" They explain their thinking to others and respond to others' thinking.
4	**Model with mathematics**
	Mathematically proficient students experiment with representing problem situations in multiple ways including numbers, words (mathematical language) drawing pictures, using objects, acting out, making chart, list, or graph, creating equations etc…Students need opportunities to connect different representations and explain the connections. They should be able to use all of the representations as needed. **Third** graders should evaluate their results in the context of the situation and reflect whether the results make any sense.
5	**Use appropriate tools strategically**
	Third graders should consider all the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For example, they might use graph paper to find all possible rectangles with the given perimeter. They compile all possibilities into an organized list or a table, and determine whether they all have the possible rectangles.
6	**Attend to precision**
	Mathematical proficient **third** graders develop their mathematical communication skills; they try to use clear and precise language in their discussions with others and in their own reasoning. They are careful about specifying their units of measure and state the meaning of the symbols they choose. For instance, when figuring out the area of a rectangle the record their answer in square units.
7	**Look for and make use of structure**

	In **third** grade, students should look closely to discover a pattern of structure. For example, students' properties of operations as strategies to multiply and divide. (commutative and distributive properties.
8	**Look for and express regularity in repeated reasoning**
	Mathematically proficient students in third **grade** should notice repetitive actions in computation and look for more shortcut methods. For example, students may use the distributive property as a strategy for using products they know to solve products that they don't know. For example, if students are asked to find the product of 7x8, they might decompose 7 into 5 and 2 and then multiply 5 x 8 and 2 x 8 to arrive at 40 + 16 or 56. In addition, third graders continually evaluate their work by asking themselves, "Does this make sense?"

Effective Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions	
Practice	Description/ Questions
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task? How do you respond to the work that students are likely to produce? Which strategies from student work will be most useful in addressing the mathematical goals?
2. Monitoring	Paying attention to what and how students are thinking during the lesson. Students working in pairs or groups Listening to and making note of what students are discussing and the strategies they are using Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)
3. Selecting	This is the process of deciding the *what* and the *who* to focus on during the discussion.
4. Sequencing	What order will the solutions be shared with the class?
5. Connecting	Asking the questions that will make the mathematics explicit and understandable. Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.

3rd and 4th Grade Ideal Math Block

Essential Components

FLUENCY: Partner/Small Group
5 min.

CONCRETE, PICTORIAL, and **ABSTRACT** approaches to support **ARITHMETIC FLUENCY** and **FLUENT USE OF STRATEGIES.**

LAUNCH: Whole Group
15-20 min.

Anchor Task: Math in Focus Learn

EXPLORATION: Partner / Small Group

Math in Focus Hands-On, Guided Practice, Let's Explore

INDEPENDENT PRACTICE: Individual

Math in Focus Let's Practice, Workbook, Reteach, Extra Practice, Enrichment

MATH WORKSTATIONS: Pairs / Small Group/ Individual
15-20 min.

DIFFERENTIATED activities designed to **RETEACH, REMEDIATE, ENRICH** student's understanding of concepts.

Small Group Instruction	Technology Lab	Problem Solving Lab	Fluency Lab	Math Journal Lab

SUMMARY: Whole Group

Lesson Closure: Student Reflection; Real Life Connections to Concept

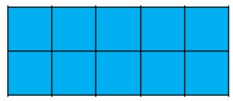
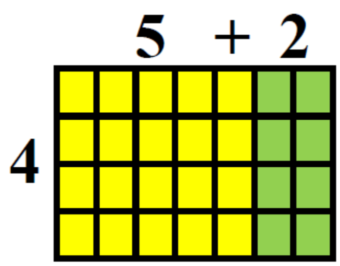
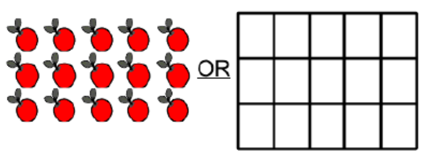
EXIT TICKET (DOL): Individual
5 min.

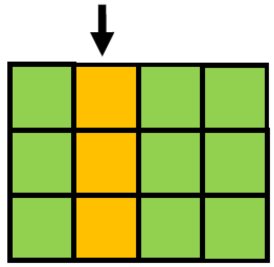
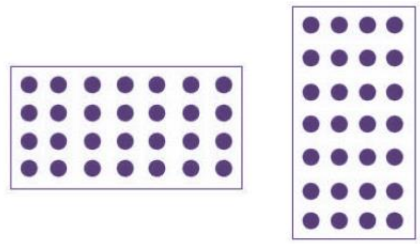
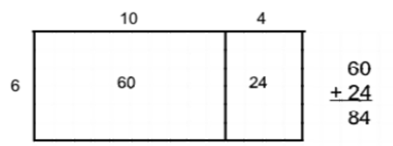
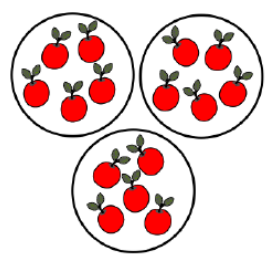
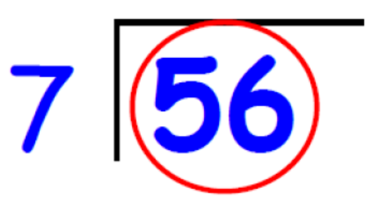
Students complete independently; Used to guide instructional decisions; Used to set instructional goals for students;

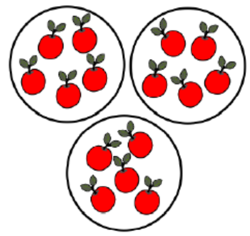
Note:

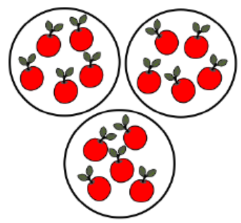
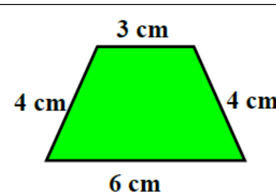
- Place emphasis on the flow of the lesson in order to ensure the development of students' conceptual understanding.
- Outline each essential component within lesson plans.
- Math Workstations may be conducted in the beginning of the block in order to utilize additional support staff.
- Recommended: 5-10 technology devices for use within **TECHNOLOGY** and **FLUENCY** workstations.

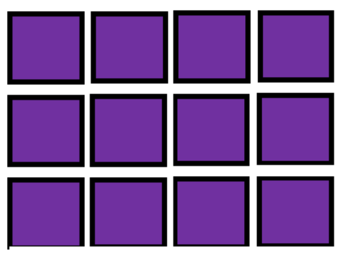
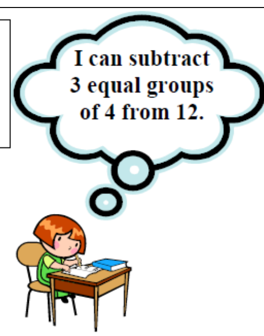
Visual Vocabulary

Visual Definition **The terms below are for teacher reference only and are not to be memorized by students.** Teachers should first present these concepts to students with models and real life examples. Students should understand the concepts involved and be able to recognize and/or use them with words, models, pictures, or numbers.		
area	**2 rows of 5 = 10 square units** **or** **2 x 5 = 10 square units** 	The measure, in square units, of the inside of a plane figure.
area model	 4 x 7 = (4 x 5) + (4 x 2) = 28	A model of multiplication that shows the product within a rectangle drawing. Can break apart the model into smaller arrays to find unknown facts.
array	**3 rows of 5** **3 x 5** 	An arrangement of objects in equal rows.
Associative Property of Multiplication	**(5 x 7) x 3 = 5 x (7 x 3)** **35 x 3 = 5 x 21** **105 = 105**	Changing the grouping of three or more factors does not change the product.

column	 **Columns go up and down.**	A vertical arrangement of numbers or information in an array or table.
Commutative Property of Multiplication	 $4 \times 7 = 7 \times 4$	Changing the order of the factors does not change the product.
Distributive Property	 6 x 14 = 6 x (10 + 4) *Break up the 14 into 10 + 4 6 x (10 + 4) (6 x 10) + (6 x 4) 60 + 24 = 84	When one of the factors of a product is a sum, multiplying each addend before adding does not change the product.
divide	 $15 \div 3 = 5$	To separate into equal groups and find the number in each group or the number of groups.
dividend		A number that is divided by another number.

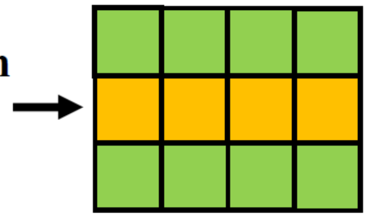
divisor	$7\overline{)56}$	The numbe which anot number is div
equal groups	 **There are 3 equal groups of 5.**	Groups that contain the same number of objects. Whenever you divide, you separate items into equal groups.
fact family	**Fact Family for 3, 5, 15** 3 x 5 = 15 15 ÷ 5 = 3 5 x 3 = 15 15 ÷ 3 = 5	A group of related facts that use the same numbers. (also known as related facts)
factor	**2 x 6 = 12** **factors**	The whole numbers that are multiplied to get a product.
inverse operations	**Multiplication and division are inverse operations.** 8 x 5 = 40 40 ÷ 5 = 8	Operations that undo each other.

multiple	12 is a multiple of 3 (and of 4) because 3 x 4 = 12	A product of a given whole number and any other whole number.
Multiplicative Identity Property of 1	 1 group of 3 = 3 1 x 3 = 3	If you multiply a number by one, the product is the same as that number.
multiply	 3 x 5 = 5 + 5 + 5	The operation of repeated addition of the same number.
partitive division (sharing division)	 Justin has 12 balloons. He wants to share them evenly among 3 friends. How many balloons should he give each friend? 12 ÷ 3 = 4	A division problem where the number of objects in each group is unknown. *How many in each group?*
perimeter	 Perimeter = 4cm + 6cm + 4cm + 3cm = 17cm	The distance around a figure.

product	$5 \times 3 = 15$	The answer to a multiplication problem.
quotative division **(measurement division)**	 Justin has 12 balloons. If he gives 3 balloons to each friend, how many friends will get balloons? $12 \div 3 = 4$	A division problem where the number of groups is unknown. *How many groups?*
quotient	$\begin{array}{r} 8 \\ 7\overline{)56} \end{array}$	The answer to a division problem.
repeated addition	 $4 + 4 + 4 = 12$	Adding equal groups of objects to find the total amount of objects.
repeated subtraction	$12 - 4 = 8$ $8 - 4 = 4$ $4 - 4 = 0$ 	Subtracting equal groups to find the total amount of groups.

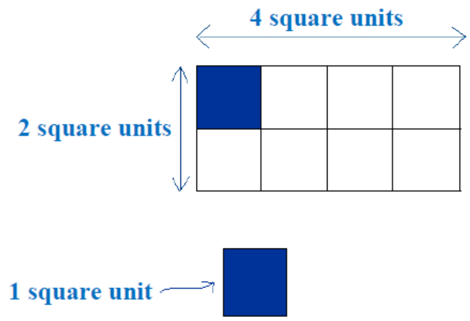
row

Rows go from left to right.



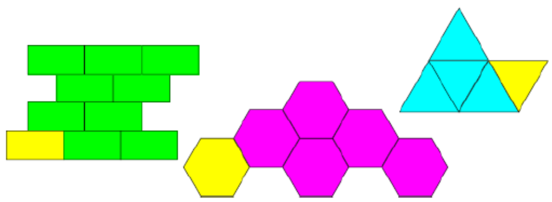
A horizontal arrangement of numbers or information in an array or table.

square unit



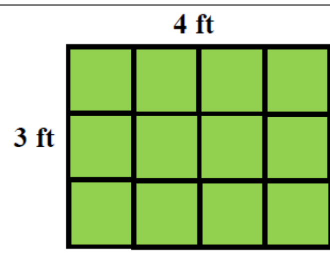
A unit, such as square centimeter or square inch, used to measure area.

tiling



A pattern of shapes repeated to fill a plane. The shapes do not overlap and there are no gaps.

width



Length	Width	Area
3	4	12 sq ft

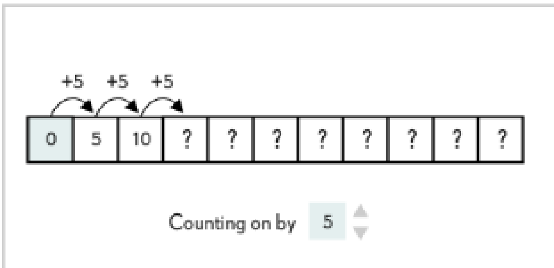
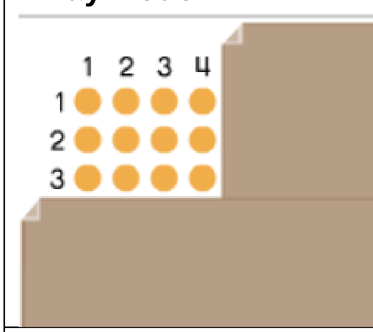
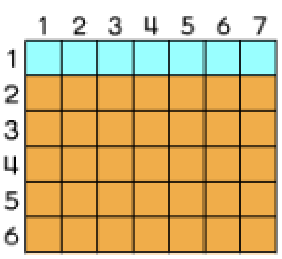
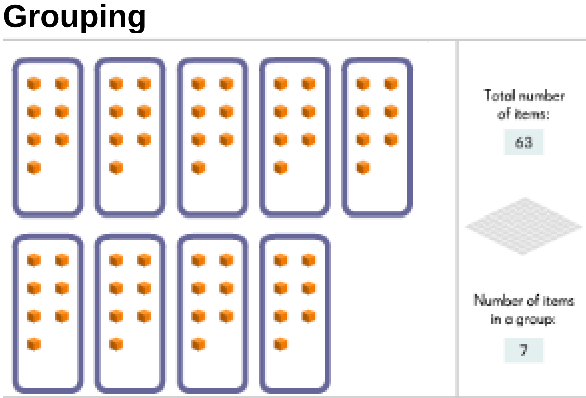
One dimension of a 2-dimensional or 3-dimensional figure.

Zero Property of Multiplication

$8 \times 0 = 0$

The product of any number and zero is 0.

Multiple Representations Framework

<table>
<tr><th colspan="2">Concrete and Pictorial Representations</th></tr>
<tr><td>Number Tape

</td><td>to multiply by skip counting</td></tr>
<tr><td>Array Model

</td><td>To show multiplication facts for numbers 1-10</td></tr>
<tr><td>Area Model

</td><td>To show multiplication facts for numbers 1-10</td></tr>
<tr><td>Grouping

</td><td>To show division by grouping</td></tr>
</table>

Unit 2 Assessment / Authentic Assessment Framework				
Assessment	**NJSLS**	**Estimated Time**	**Format**	**Graded**
Chapter 6				
Optional Chapter 6 Test/Performance Task	3.OA.4-7, 3.OA.9, 3.MD.6	1 block	Individual	Yes
Authentic Assessment #5	3.OA.4	½block	Individual	Yes
Chapter 7				
Optional Chapter 7 Test/Performance Task	3.NBT.3, 3.OA3-5, 3.OA.7 3.OA.9	1 block	Individual	Yes
Chapter 8				
Optional Chapter 8 Test/Performance Task	3.OA4-7, 3.OA.9	1 block	Individual	Yes
Chapter 9				
Optional Chapter 9 Test/Performance Task	3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7, 3.OA.8	1 block	Individual	Yes
Authentic Assessment #6	3.OA.5, 3.OA.6	½ block	Individual	Yes
Eureka Module 4				
Optional End of Module Assessment	3.MD.5-3.MD.7	1 block	Individual	Yes
Authentic Assessment #7	3.MD.7	½ block	Individual	Yes

	PLD	**Genesis Conversion**
Rubric Scoring	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

Authentic Assessment #5 – Finding the Unknown in a Division Equation

Name:____

Tehya and Kenneth are trying to figure out which number could be placed in the box to make this equation true.

Tehya insists that 12 is the only number that will make this equation true.

Kenneth insists that 3 is the only number that will make this equation true.

$$2 = \square \div 6$$

Who is right? Why? Draw a picture to support your idea.

Authentic Assessment #5 Scoring Rubric – Finding the Unknown in a Division Equation

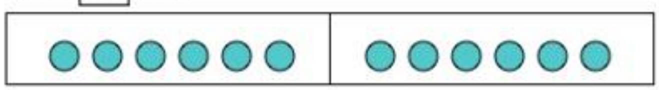
3.OA.4: Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = _ ÷ 3, 6 × 6 = ?*

Mathematical Practice: 1 and 2

SOLUTION:

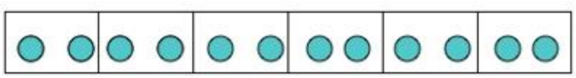
This solution shows that 12 split into groups of 6 will result in 2 groups.

$2 = \boxed{12} \div 6$



This solution shows that 12 split into 6 equal groups will result in 2 in each group.

$2 = \boxed{12} \div 6$

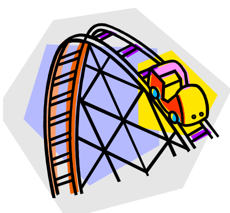


Level 5: Distinguished Command	**Level 4: Strong Command**	**Level 3: Moderate Command**	**Level 2: Partial Command**	**Level 1: No Command**
Clearly constructs and communicates a complete response based on explanations/reasoning using (the): • Visual representations • relationship between multiplication and division Response includes an **<u>efficient</u>** and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/reasoning using (the): • Visual representations • relationship between multiplication and division Response includes a **<u>logical</u>** progression of steps	Clearly constructs and communicates a complete response based on explanations/reasoning using (the): • Visual representations • relationship between multiplication and division Response includes a **<u>logical but incomplete</u>** progression of steps. Minor calculation errors.	Clearly constructs and communicates a complete response based on explanations/reasoning using (the): • Visual representations • relationship between multiplication and division Response includes an **<u>incomplete or Illogical</u>** progression of steps.	The student shows no work or justification

Authentic Assessment #6 – Amusement Park

Name:_______________________________________

Juan is having his birthday party at the amusement park. He and his friends have broken up into two equal groups of four, so that their parents can chaperone them easily. His mom has bought a total of 72 ride tickets for Juan and each of his friends. How many tickets will each group get? How many tickets will each child get? Use pictures, mathematical operations, and words to explain your answer.



Authentic Assessment #6 Scoring Rubric – Amusement Park

3.OA.5: Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*

3.OA.6: Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8*.

Mathematical Practice:1, 3, 6

Type: Individual, Individual w/Interview

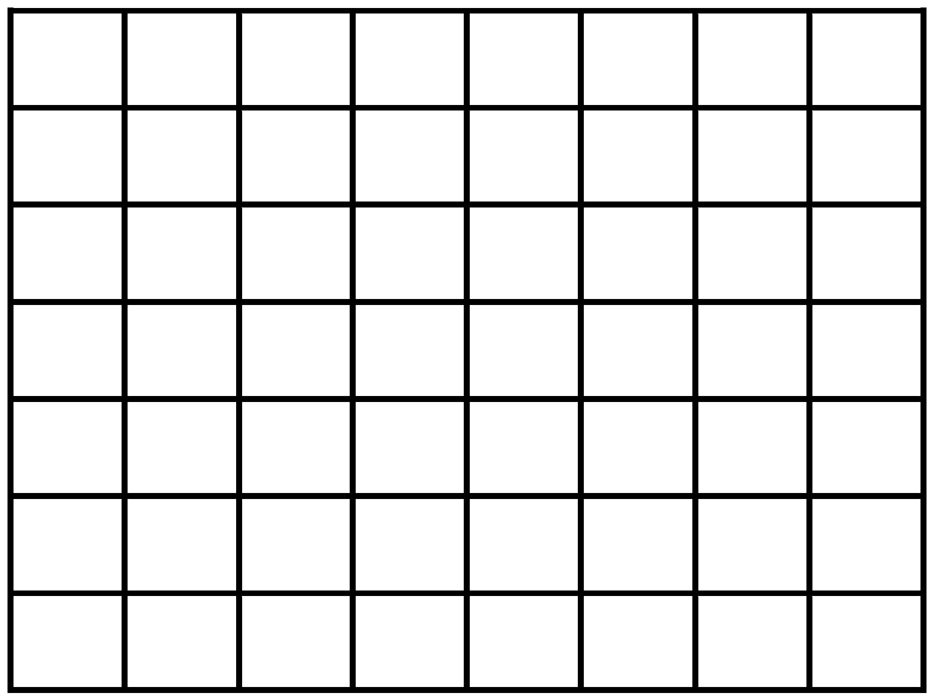
<table>
<tr><td colspan="5">SOLUTION:

See below</td></tr>
<tr><td>Level 5:
Distinguished Command</td><td>Level 4:
Strong Command</td><td>Level 3:
Moderate Command</td><td>Level 2:
Partial Command</td><td>Level 1:
No Command</td></tr>
<tr><td colspan="2">Student has an understanding of multiplication and division. Student correctly determines the amount of children (including Juan) to be 8. In addition, the student correctly identifies the total number of tickets needed to be is 72. The student correctly determines the amount of tickets each group gets is 36 (72 total tickets divided by 2 groups). The student then identifies that the amount of tickets per group (36) must be divided by the amount of people in each group (36/4). The students identifies that each person will get 9 tickets in each group (9x4 = 36). All of the information and explains his/her conclusion through the use of mathematical language, pictures and diagrams, and/or mathematical processes.</td><td>Student has an understanding of multiplication and division, however the student does not identify each the amount of tickets each student is to receive. Student has an understanding of dividing the amount of tickets (72) by 2 for the 2 groups, however does not identify what each student should get. The student shows his/her work, however, has limited explanation through</td><td>Student may determine how many children are at the party, but fails to figure out the total number of tickets that are needed. The student does not show work and has flaws in their approach to answer the problem.</td><td>Does not address task, unresponsive, unrelated or inappropriate.</td></tr>
</table>

		the use of language, pictures, diagrams, and/or mathematical processes.		
Clearly constructs and communicates a complete response based on explanations/reasoning using the: • Properties based on place value • properties of operations • relationship between addition and subtraction Response includes an **<u>efficient</u>** and logical progression of steps.	Clearly constructs and communicates a complete response based on explanations/ reasoning using the: • Properties based on place value • properties of operations • relationship between addition and subtraction Response includes a **<u>logical</u>** progression of steps	Constructs and communicates a complete response based on explanations/ reasoning using the: • Properties based on place value • properties of operations • relationship between addition and subtraction Response includes a **<u>logical but incomplete</u>** progression of steps. Minor calculation errors.	Constructs and communicates an incomplete response based on explanations/ reasoning using the: • properties of operations • relationship between addition and subtraction Response includes an **<u>incomplete or Illogical</u>** progression of steps.	The student shows no work or justification

Authentic Assessment #7 Micah and Nina's Rectangle

Micah and Nina want to determine the area of this rectangle.



Micah found the rectangle's area using the following equation: **8 x 7 =** ***a***.

Nina found the area by adding the products of the following equations:

2 x 7 = ***a*** and**5 x 7 =** ***b***.

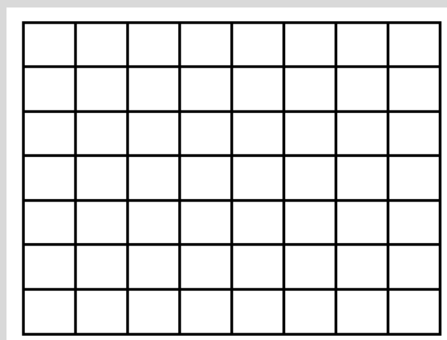
Whose equation(s) will find the correct area of the rectangle? Explain.

What other strategy can be used to find the area of this rectangle?

Micah and Nina’s Rectangle

3.MD.7– Task 2

<table>
<tr><td>Standard(s)</td><td>3.MD.7 Relate area to the operations of multiplication and addition.
3.MD.7aFind the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
3.MD.7cUse tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
3.MD.7dRecognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.</td></tr>
<tr><td>Materials</td><td>Micah and Nina’s Area Model handout (see attached), pencils, scissors (optional)</td></tr>
<tr><td>Task</td><td>Micah and Nina were trying to determine the area of this rectangle.



Micah found the rectangle’s area by adding the products of the following equation: 8x5=a and 7x8=b.
Nina found the area by adding the products of the following equations: 2x7=a and6x7=b.
For each student, calculate the total area.
Is each correct? Explain why or why not.
Write a sentence to explain what other strategy can be used to find the area of this rectangle.</td></tr>
</table>

Performance Task Scoring Rubric #7:

Level 5: Distinguished Command	Level 4: Strong Command	Level 3: Moderate Command	Level 2: Partial Command	Level 1: No Command
Student gives all 5 correct answers. Clearly constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction relationship Response includes an **efficient** and logical progression of steps.	**Student gives all 5 correct answers.** Clearly constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction • relationship between multiplication and division Response includes a **logical** progression of steps	**Student gives all 4 correct answers.** Constructs and communicates a complete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction • relationship between multiplication and division Response includes a **logical but incomplete** progression of steps. Minor calculation errors.	**Student gives 3 correct answers.** Constructs and communicates an incomplete response based on explanations/reasoning using the: • properties of operations • relationship between addition and subtraction • relationship between multiplication and division Response includes an **incomplete or illogical** progression of steps.	**Student gives less than 3 correct answers.** The student shows no work or justification.

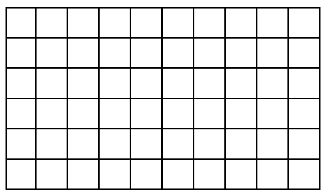
Additional Resources

3.MD.7 Lesson 1

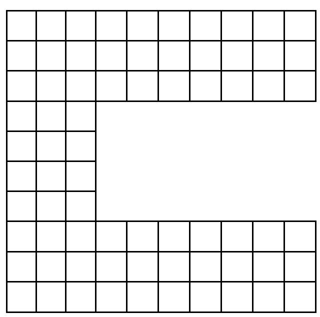
Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

Imagine that each square in the picture measures one centimeter on each side. What is the area of each shape? Try to work it out without counting each square individually.

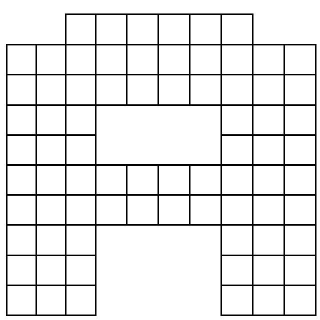
1.



2. Decompose the object below in to rectangles to find the area of the entire object.



3. Decompose the object below in to rectangles to find the area of the entire object.



Focus Questions

Question 1: Show how you divided each object to find the area.
Question 2: How do the squares covering a rectangle compare to an array?

Journal Question

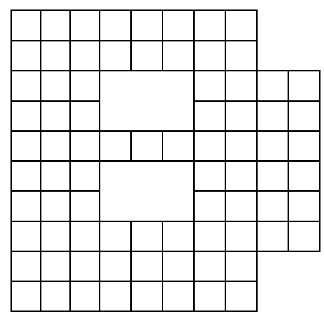
Is a square a rectangle?
Why or why not?

3.MD.7.a: Lesson 1

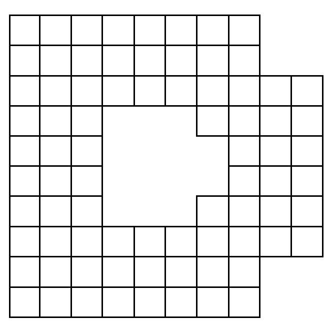
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

4. Decompose the object below in to rectangles to find the area of the entire object.



5. Decompose the object below in to rectangles to find the area of the entire object.



3.MD.7.a: Lesson 1

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

Name________________________ **Date________________**

Finding Area Using Square Units

Find the area of each figure. A quick hint is to rearrange the composition of each figure to make a shape you can work with.

3.MD.7.a: Lesson 1

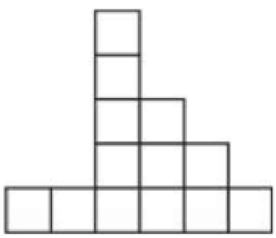
Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

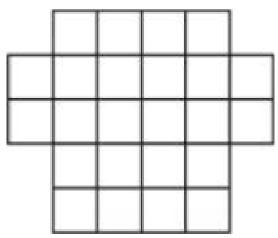
Name ______________________________ Date ______________________

Finding Area Using Square Units

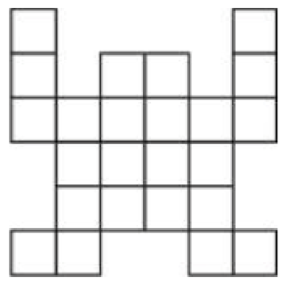
Find the area of each figure. A quick hint is to rearrange the composition of each figure to make it a shape you can work with.



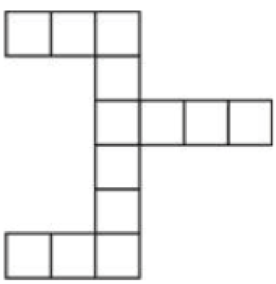
1. Area = _____ Square units



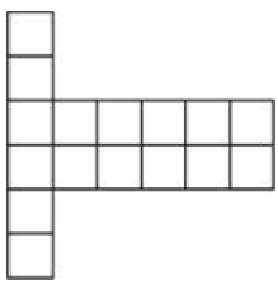
2. Area = _____ Square units



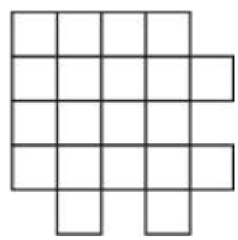
3. Area = _____ Square units



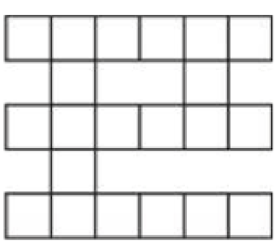
4. Area = _____ Square units



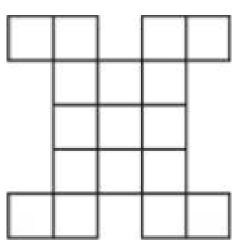
5. Area = _____ Square units



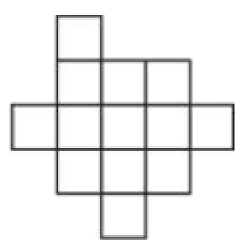
6. Area = _____ Square units



7. Area = _____ Square units

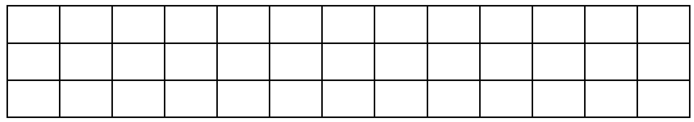


8. Area = _____ Square units



9. Area = _____ Square units

10. Tanya built this rectangular model using 39 tiles.



a. List two number sentences this model represents.

b. Tanya found one more tile. Draw a new rectangular model using all of Tanya’s tiles.

c. List two multiplication number sentences this new model represents.

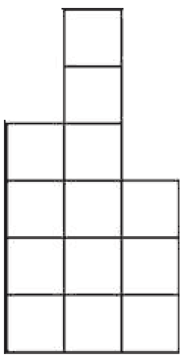
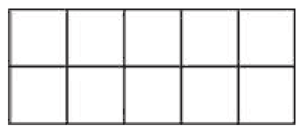
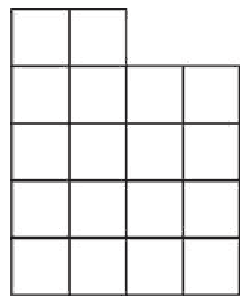
3.MD.7.a: Lesson 1

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

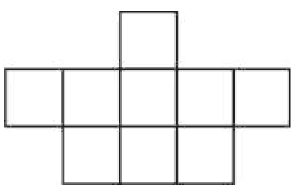
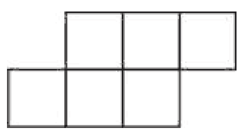
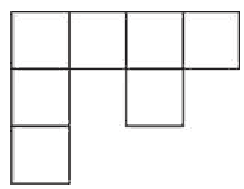
Name________________________ **Date________________**

Area of Unusual Shapes with Square Units

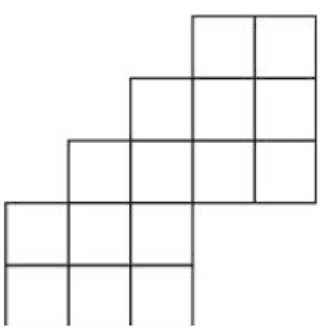
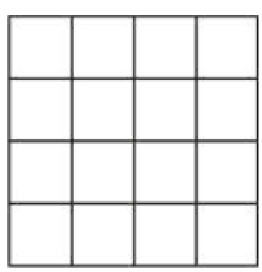
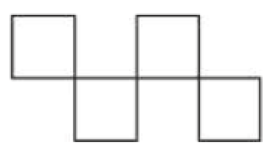
Find the area of each figure. A quick hint is to rearrange the composition of each figure to make a shape you can work with.

1. Area = _____ Square units 2. Area = _____ Square units 3. Area = _____ Square units

4. Area = _____ Square units 5. Area = _____ Square units 6. Area = _____ Square units

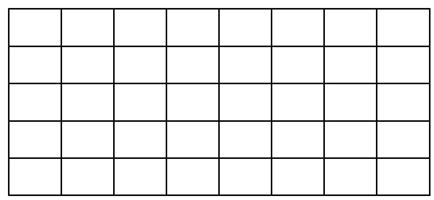
7. Area = _____ Square units 8. Area = _____ Square units 9. Area = _____ Square units

3.MD.7.a: Lesson 1

Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

10. Amanda wants to cover the top of her doll's table with colored paper. The top of the table is shown below.

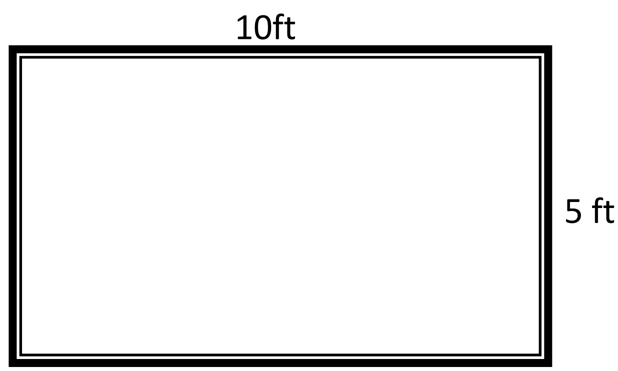


How many square centimeters of paper does Amanda need if each square equals 1 square centimeters?

Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

Below is the floor plan for Paul's kitchen. How many square foot tiles will he need to cover the floor?



Focus Questions

Question 1: What strategies can be used to find the area of a shape?
Question 2: How is multiplication related to finding area?

Journal Question

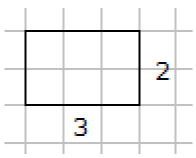
How would you explain how to find area to a second grader?

3.MD.7.b: Lesson 2

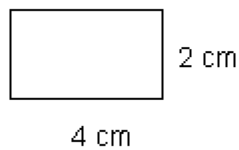
Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

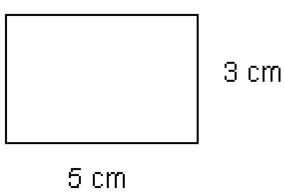
Name ________________________ **Date** ___________________



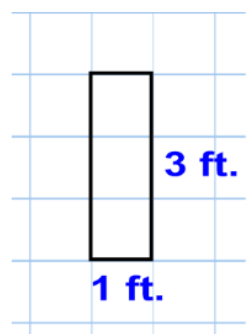
1. Area = _____ Square units



2. Area = _____ Square cm



3. Area = _____ Square cm



4. Area = _____ Square ft

5. What is the area of a rectangle with side length of 5 inches and a side width of 8 inches?

Number sentence: ____

6. What is the area of a rectangle with the side length of 7 feet and a side width of 3 feet?

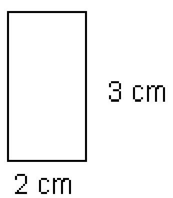
Number sentence: ____

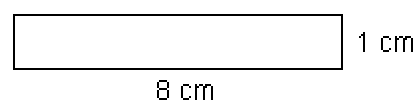
3.MD.7.b: Lesson 2

Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

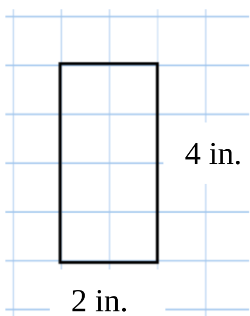
Name ________________________ **Date** ___________________

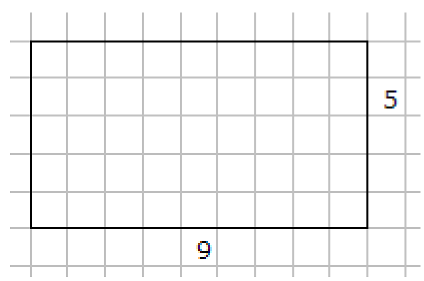




1. Area = _____ Square cm

2. Area = _____ Square cm





3. Area = _____ Square inches

4. Area = _____ Square units

5. What is the area of a rectangle with side length of 6 meters and a side width of 7 meters?

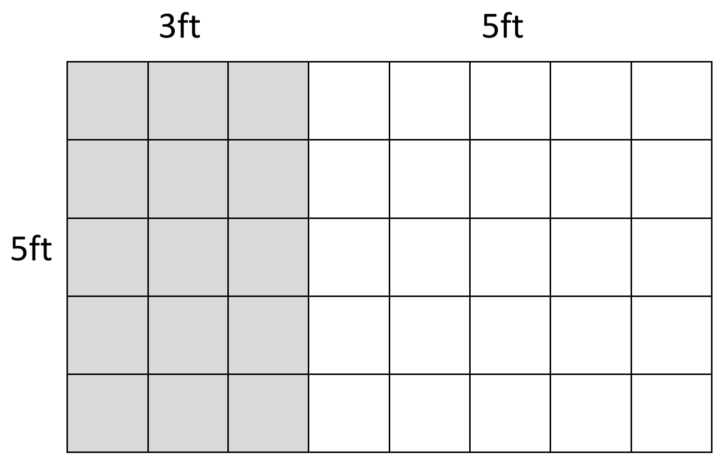
Number sentence: ____

6. What is the area of a rectangle with the side length of 5 feet and a side width of 2 feet?

Number sentence: ____

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

Joe and John are installing windows in their new home. The first window is 5' by 3' and the second window is 5' by 5'. They are placing the windows in the wall side-by-side so that there was no space between them. How much area will the two windows cover?



Focus Questions

Question 1: Can you write an equation for the situation above?
Question 2: Is there a simpler way to find the area that the two windows will cover?
Question 3: Can you write an equation for Question 2?

Journal Question

What do you think distributing has to do with the distributive property?

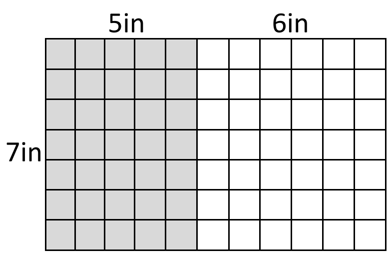
3.MD.7.c: Lesson 3

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and*b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

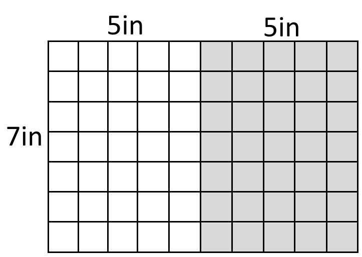
Directions: Without counting, show two ways of finding the area of each object.

1.



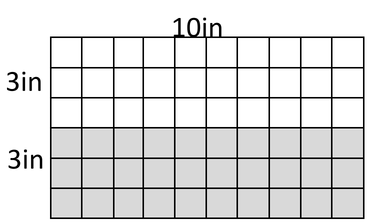
Area = _____ Square in

2.



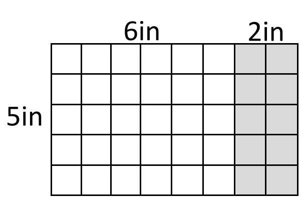
Area = _____ Square in

3.



Area = _____ Square in

4.



Area = _____ Square in

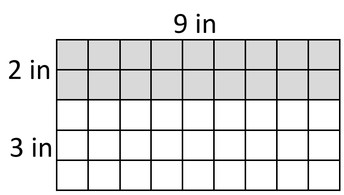
3.MD.7.c: Lesson 3

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and$b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

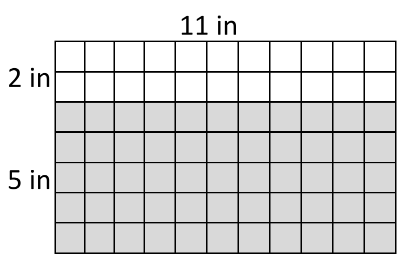
Directions: Without counting, show two ways of finding the area of each object.

5.



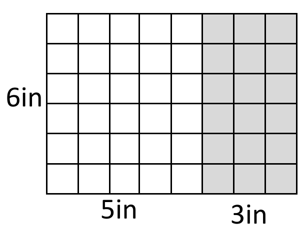
Area = _____ Square in

6.



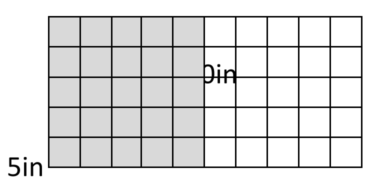
Area = _____ Square in

7.



Area = _____ Square in

8.



Area = _____ Square in

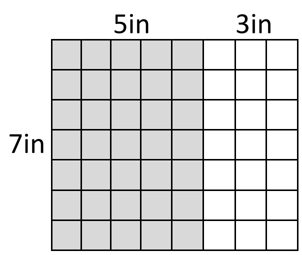
3.MD.7.c: Lesson 3

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment

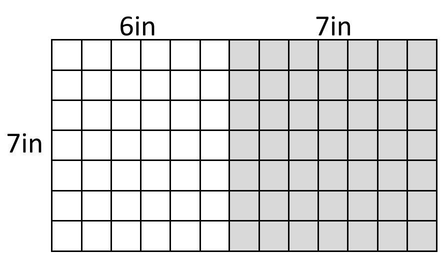
Directions: Without counting, show two ways of finding the area of each object.

1.



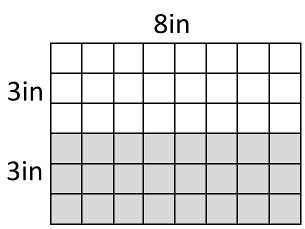
Area = _____ Square in

2.



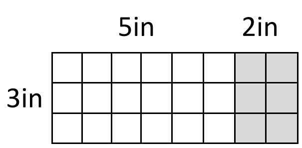
Area = _____ Square in

3.



Area = _____ Square in

4.



Area = _____ Square in

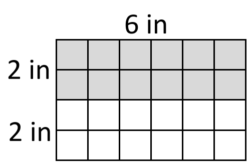
3.MD.7.c: Lesson 3

Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and*b* + *c* is the sum of *a* × *b* and *a* × *c*. Use area models to represent the distributive property in mathematical reasoning.

Introductory Task	Guided Practice	Collaborative	Homework	Assessment

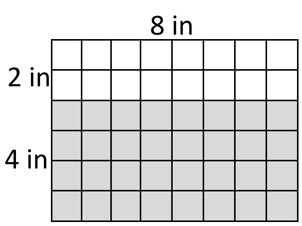
Directions: Without counting, show two ways of finding the area of each object.

5.



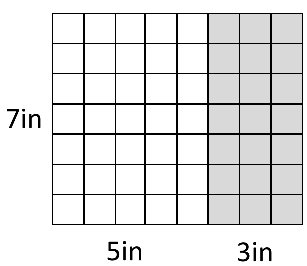
Area = _____ Square in

6.



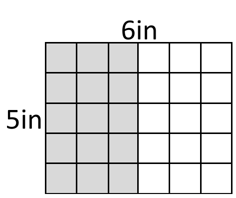
Area = _____ Square in

7.



Area = _____ Square in

8.



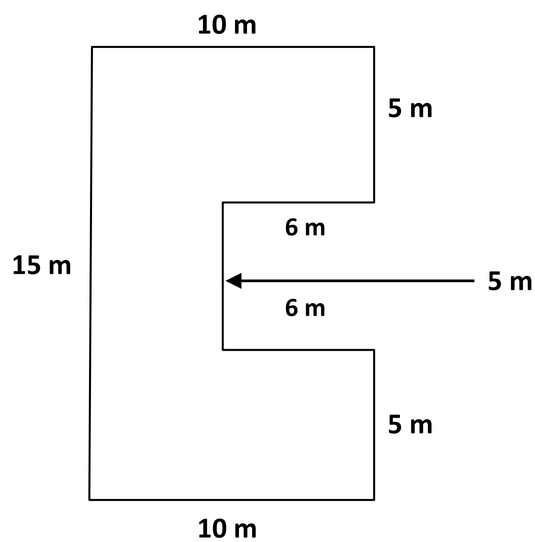
Area = _____ Square in

3.MD.7.d: Lesson 4

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

A storage shed is pictured below. What is the total area? How could the figure be decomposed to help find the area?



Focus Questions

Question 1: How can decomposing a figure into smaller figures help solve complex math problems?
Question 2: How do multiplication equations help solve area problems?

Journal Question

How can decomposing diagrams help you answer multiplication problems?

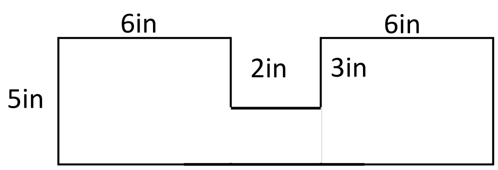
3.MD.7.d: Lesson 4

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Introductory Task	***Guided Practice***	***Collaborative***	***Homework***	***Assessment***

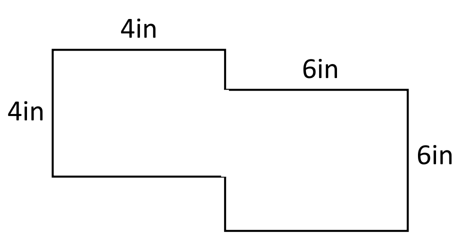
Decompose the figure to find the total area of each figure.

1.



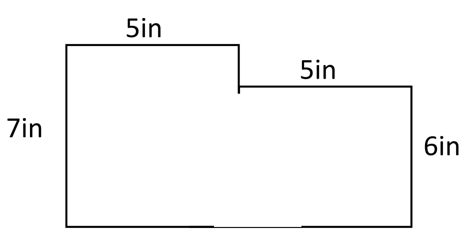
Area = _____ Square in

2.



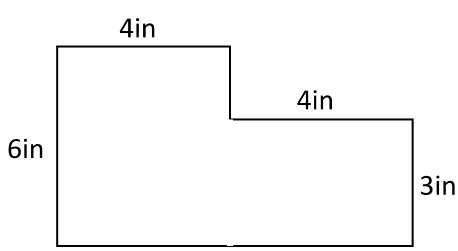
Area = _____ Square in

3.



Area = _____ Square in

4.



Area = _____ Square in

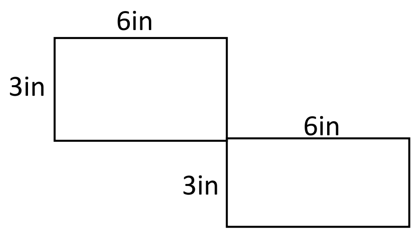
3.MD.7.d: Lesson 4

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

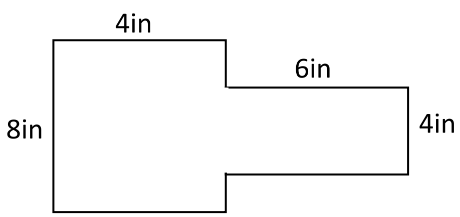
Decompose the figure to find the total area of each figure.

5.



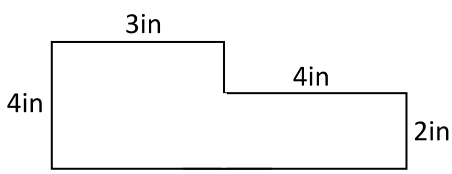
Area = _____ Square in

6.



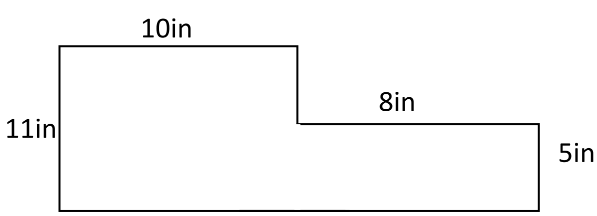
Area = _____ Square in

7.



Area = _____ Square in

8.



Area = _____ Square in

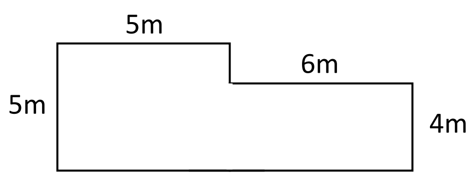
3.MD.7.d: Lesson 4

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Introductory Task	*Guided Practice*	*Collaborative*	*Homework*	*Assessment*

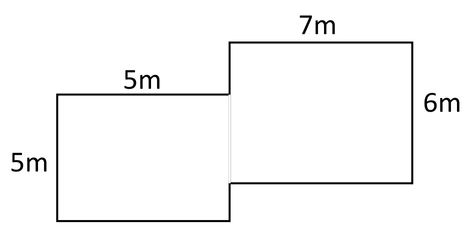
Decompose the figure to find the total area of each figure.

1.



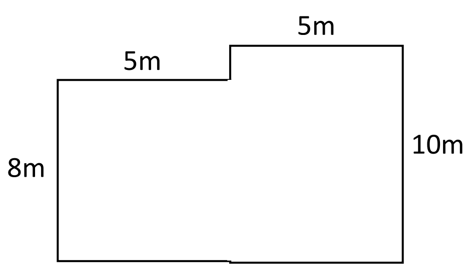
Area = _____ Square m

2.



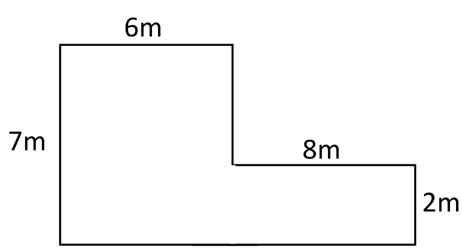
Area = _____ Square m

3.



Area = _____ Square m

4.



Area = _____ Square m

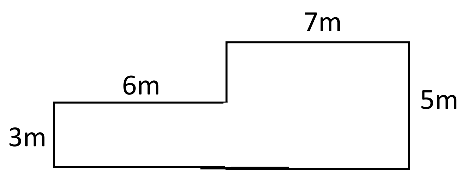
3.MD.7.d: Lesson 4

Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Introductory Task	*Guided Practice*	*Collaborative*	***Homework***	*Assessment*

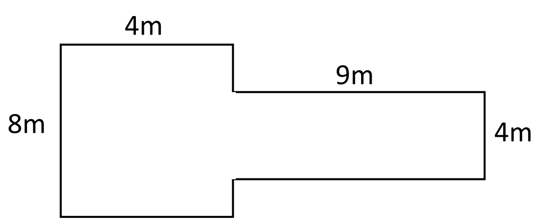
Decompose the figure to find the total area of each figure.

5.



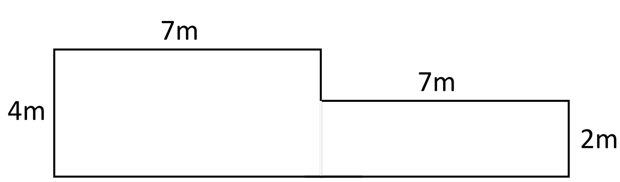
Area = _____ Square m

6.



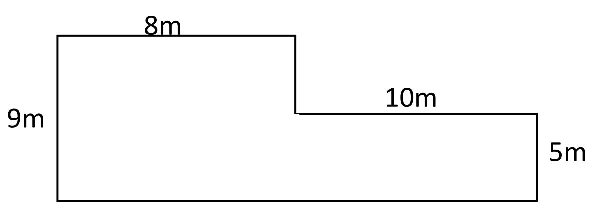
Area = _____ Square m

7.



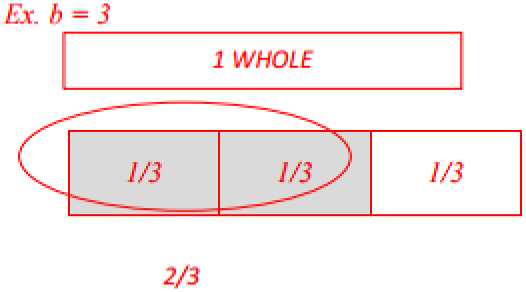
Area = _____ Square m

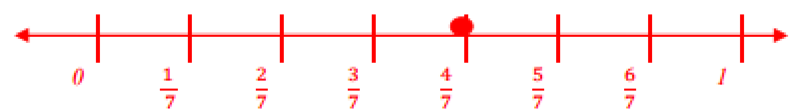
8.



Area = _____ Square m

NJDOE 3rd -5th Grade Mathematics Revisions

Grade level	Standard	Revised Standard
3	3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 × 7.	3.OA.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. For example, describe and/or represent a context in which a total number of objects can be expressed as 5 × 7.
3	3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	3.OA.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe and/or represent a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.
3	3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b	3.NF.1 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. *Ex. b = 3* 
3	3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a.	3.NF.2 Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a

	Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. *Ex. a = 4; b = 7* 
3	3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	3.MD.6 Measure areas by counting unit squares (square cm, square m, square in, square ft, and non-standard units).
4	4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two - column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36),	4.MD.1 Know relative sizes of measurement units within one system of units including km, m, cm, mm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...
5	5.MD.5b. Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems	5.MD.5b Apply the formulas V = l × w × h and V = B × h for rectangular prisms to find volumes of right rectangular prisms with whole- number edge lengths in the context of solving real world and mathematical problems
5	5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	5.MD.4 Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and non-standard units.

21st Century Career Ready Practices

CRP1. Act as a responsible and contributing citizen and employee.

CRP2. Apply appropriate academic and technical skills.

CRP3. Attend to personal health and financial well-being.

CRP4. Communicate clearly and effectively and with reason.

CRP5. Consider the environmental, social and economic impacts of decisions.

CRP6. Demonstrate creativity and innovation.

CRP7. Employ valid and reliable research strategies.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

CRP9. Model integrity, ethical leadership and effective management.

CRP10. Plan education and career paths aligned to personal goals.

CRP11. Use technology to enhance productivity.

CRP12. Work productively in teams while using cultural global competence.